On optimal error rates for strong approximation of SDEs with non-Lipschitzian drift

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im Raum 147b, JUR, Innstr. 39, Universität Passau, 94032 Passau
um 14:50 Uhr
von Herrn Simon Ellinger
Betreuung: Prof. Dr. Thomas Müller-Gronbach und
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Motivated by applications in financial mathematics and energy markets, the interest in solutions of stochastic differential equations (SDEs) with non-Lipschitz continuous coefficients has grown in recent years. This raises the question of which is the best method for approximating solutions of such SDEs. For example, it has been shown that in the case of piecewise Lipschitz continuous coefficients, a transformed Milstein scheme converges to the solution at the final time point with a rate of at least $3/4$ in terms of number of evaluations of the driving Brownian motion. We show that the rate $3/4$ is the best possible error rate for final time approximations that can be achieved by any method based on finitely many evaluations of the Brownian motion for a large class of piecewise Lipschitz continuous coefficients, extending a result under more restrictive assumptions on the coefficients.

In addition to piecewise Lipschitz continuous coefficients, SDEs with coefficients that are α-Hölder continuous or Sobolev regular of order α were also investigated. In this case, it is known that the Euler-Maruyama scheme converges to the solution at the final time point with a rate of at least $(1+\alpha)/2$, up to some small $\varepsilon \in (0,\infty)$. We show that this rate cannot be improved in general.

For the derivation of the lower bounds we use a new proof technique, namely the local coupling of noise technique. Therewith, we reduce the analysis of the error of an arbitrary approximation method based on finitely many evaluations of the Brownian motion to the analysis of the distance of two occupation time functionals on an interval between two consecutive discretization points.