Diophantine Conditions and the Asymptotic Behavior of Lacunary Sums

Doktorandenkolloquium am Mittwoch, 29.05.2024
im HS 13, IM, Innstr. 33, Universität Passau, 94032 Passau
um 16:05 Uhr
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Betreuung: Prof. Dr. Joscha Prochno

We consider a sequence of natural numbers $(n_k)_{k \in \mathbb{N}}$ which satisfies the condition

$$\inf_{k \in \mathbb{N}} \frac{n_{k+1}}{n_k} > 1,$$

a 1-periodic function $f : \mathbb{R} \to \mathbb{R}$, and a random variable $x \sim \mathrm{Unif}([0, 1])$. The associated quantity

$$S_N(x) := \sum_{k=1}^{N} f(n_k x), \qquad N \in \mathbb{N},$$

is referred to as a lacunary sum. Our goal is to study classical L_2-limit theorems, such as central limit theorems (CLTs) and laws of the iterated logarithm (LILs), for $S_N(x)$ as $N \to \infty$. Aistleitner and Berkes made significant progress in [2], demonstrating that the presence of not too many solutions to the set of 2-term Diophantine equations

$$an_k + bn_l = c, \qquad 1 \le k, l \le N, \tag{1}$$

where $a, b, c \in \mathbb{Z}$, ensures the existence of a CLT with the same variance as the CLT obtained by a sum of certain i.i.d. random variables. Shortly thereafter, Aistleitner showed in [1] that a stronger Diophantine condition, as required for the CLT, ensures that $S_N(x)$ satisfies an LIL which is in accordance with the LIL for truly independent random variables. Recently, in an article together with Aistleitner and Prochno (see [3]) we showed the remarkable result that the stronger Diophantine condition for the LIL is indeed necessary. In an ongoing project with C. Aistleitner and J. Prochno, we aim to further understand the interplay between the tail behaviour of $S_N(x)$ as $N \to \infty$ and the number of Diophantine solutions to (1).

References

[1] Christoph Aistleitner. On the law of the iterated logarithm for the discrepancy of lacunary sequences. *Trans. Amer. Math. Soc.*, 362(11):5967–5982, 2010.

[2] Christoph Aistleitner and István Berkes. On the central limit theorem for $f(n_k x)$. *Probab. Theory Related Fields*, 146(1-2):267–289, 2010.

[3] Christoph Aistleitner, Lorenz Frühwirth, and Joscha Prochno. Diophantine conditions in the law of the iterated logarithm for lacunary systems. *Probab. Theory Related Fields*, pages 1–30, 2024.