Consistency of randomized integration methods

Doktorandenkolloquium am Mittwoch, 29.05.2024 im HS 13, IM, Innstr. 33, Universität Passau, 94032 Passau um 15:15 Uhr von Herrn Julian Hofstadler Betreuung: Prof. Dr. Daniel Rudolf

An important task in many areas of modern statistics, or science in general, is the computation of integrals, for instance if one wants to know the expectation of some random variable. It is a common situation that an integral of interest cannot be evaluated exactly, e.g. if the integrand is too complex, and in this case we rely on numerical methods such as structured Monte Carlo algorithms. In this talk we focus on the prototypical setting where the integrand of interest f is defined on the d-dimensional unit cube $[0, 1]^d$. Different algorithms are available in this situation, for example scrambled (t, d) sequences, Latin hypercube sampling, or randomized Frolov quadrature and if the function f is integrable and satisfies some additional regularity assumptions, then we know that these algorithms converge to the quantity of interest. We present a result which ensures that a class of structured randomized integration methods consistently estimates the integral of any given $f \in L^1[0, 1]^d$, where consistency refers to convergence in mean and/or probability. As a consequence, we are able to prove consistency of different algorithms, where we only require that f is integrable without any further regularity conditions.